

POLYNOMIAL OPERATIONS

ADDITION AND SUBTRACTION:

Adding and subtracting polynomials is the same as the procedure used in combining like terms. When *adding* polynomials, simply drop the parenthesis and combine like terms. When *subtracting* polynomials, distribute the negative first, then combine like terms.

Examples:

Addition:

$$(2x^2 + 3x - 7) + (3x^2 - 4x - 10) = 2x^2 + 3x^2 + 3x - 4x - 7 - 10 = 5x^2 - x - 17$$

Subtraction:

$$(5x^2 - 12x + 1) - (2x^2 + 3x - 7) = 5x^2 - 12x + 1 - 2x^2 - 3x + 7 = 3x^2 - 15x + 8$$

MULTIPLICATION:

1. Monomial times Monomial: To multiply a monomial times a monomial, just multiply the numbers then multiply the variables using the rules for exponents.

Example:

$$(-2x^2y)(5xy^7) = -2 \cdot 5x^2 \cdot x \cdot y \cdot y^7 = -10x^3y^8$$

2. Monomial times Polynomial: Simply use the distributive property to multiply a monomial times a polynomial.

Examples:

$$\text{a. } -2x(x^2 + 3x - 8) = -2x(x^2) - 2x(3x) - 2x(-8) = -2x^3 + 6x^2 + 16x$$

$$\text{b. } 5x^2(-2x^4 + 3y - 6) = 5x^2(-2x^4) + 5x^2(3y) + 5x^2(-6) = -10x^6 + 15x^2y - 30x^2$$

3. Binomial times a Binomial: To multiply two binomials, use the **FOIL** method (**F**irst times first, **O**utside times outside, **I**nside times inside, and **L**ast times last).

Example:

$$(x + 2)(x - 3) = x(x) + x(-3) + 2(x) + 2(-3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$$

Special Products: The following formulas may be used in these special cases as a short cut to the FOIL method.

Difference of Squares:

$$(a + b)(a - b) = a^2 - b^2$$

Perfect Squares:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Example:

$$(3x + 4)(3x - 4) = 9x^2 - 16$$

Example:

$$(x + 4)^2 = x^2 + 2(x)(4) + 4^2 = x^2 + 8x + 16$$

Example:

$$(x - 3)^2 = x^2 - 2(x)(3) + 3^2 = x^2 - 6x + 9$$

4. Polynomial times polynomial: To multiply two polynomials where at least one has more than two terms, distribute each term in the first polynomial to each term in the second.

Examples:

$$\begin{aligned}
 \text{a. } (x^2 + 3x - 4)(x^2 - 6x + 5) &= x^2(x^2) + x^2(-6x) + x^2(5) + 3x(x^2) + 3x(-6x) + 3x(5) - 4(x^2) - 4(-6x) - 4(5) \\
 &= x^4 - 6x^3 + 5x^2 + 3x^3 - 18x^2 + 15x - 4x^2 + 24x - 20 \\
 &= x^4 - 3x^3 - 17x^2 + 39x - 20 \\
 \text{b. } (2x - 3)(4x^2 - 5x + 1) &= 2x(4x^2) + 2x(-5x) + 2x(1) - 3(4x^2) - 3(-5x) - 3(1) \\
 &= 8x^3 - 10x^2 + 2x - 12x^2 + 15x - 3 \\
 &= 8x^3 + 22x^2 + 17x - 3
 \end{aligned}$$

DIVISION:

1. Division by Monomial: Each term of the polynomial is divided by the monomial and it is simplified as individual fractions.

Examples:

$$\begin{aligned}
 \text{a. } \frac{3x^2 - 9x + 14}{3x} &= \frac{3x^2}{3x} - \frac{9x}{3x} + \frac{14}{3x} = x - 3 + \frac{14}{3x} \\
 \text{b. } \frac{10y^2 - 25y + 20}{-5y} &= \frac{10y^2}{-5y} - \frac{25y}{-5y} + \frac{20}{-5y} = -2y + 5 - \frac{4}{y}
 \end{aligned}$$

2. Division by Binomial or Larger Polynomial:

Use the long division format as follows:

- Both the divisor and the dividend must be written in descending order.
- Any missing powers should be replaced by zero.
- All remainders are in fraction form (remainder/divisor) and are added to the quotient.

Examples:

$$\text{a. } (x^2 - 2x - 15) \div (x + 3) = x - 5$$

$$\text{b. } \frac{9x^3 - x + 3}{3x - 2} = 3x^2 + 2x + 1 + \frac{5}{3x - 2}$$

$$\begin{array}{r}
 \overline{) x^2 - 2x - 15} \\
 \underline{-(x^2 + 3x)} \\
 -5x - 15 \\
 \underline{-(-5x - 15)} \\
 0
 \end{array}$$

$$\begin{array}{r}
 \overline{) 9x^3 + 0x^2 - x + 3} \\
 \underline{-(9x^3 - 6x^2)} \\
 6x^2 - x \\
 \underline{-(6x^2 - 4x)} \\
 3x + 3 \\
 \underline{-(3x - 2)} \\
 5
 \end{array}$$

POLYNOMIAL OPERATIONS PRACTICE

Add the following polynomials (Write answers in descending order):

1. $(7j^3 - 2) + (5j^3 - j - 3)$
2. $(8a^5 - 4) + (3a^5 + a - 2)$
3. $(6m^5 + 1) + (2m^5 + 9m - 1)$
4. $(3m^5 + 1) + (9m^5 + 3m - 2)$
5. $(-5x^2 - x + 4) + (-3x^2 - 5x + 2)$
6. $(-4x + 4x^3 + 7) + (3x^3 - 9 - 3x)$
7. $(3x^2 - 2x + 1) + (-x^2 + 3x + 1)$

Subtract the following polynomials (Write answers in descending order):

8. $(-x^2 + x - 4) - (3x^2 - 8x - 2)$
9. $(8x^2 - 3x) - (5x - 5 - 8x^2)$
10. $(-x^2 - 5x - 3) - (-7x^2 - 8x - 8)$
11. $(-2x^3 + x) - (7x - 3 - 7x^3)$
12. $(3x^3 + 3x^2 + 9) - (5x^3 - 7x^2 + 6x - 9)$
13. $(5x^3 + 5x^2 + 5) - (6x^3 - 6x^2 + 8x - 5)$
14. $(5x^3 + 3x^2 + 5) - (7x^3 - 9x^2 + 8x - 5)$

Multiply the following polynomials:

- | | |
|----------------------------------|-------------------------------|
| 15. $(8x^3y^2)(-3x^2y^3)$ | 25. $(4x - 3)(3x - 5)$ |
| 16. $(-9x^3y)(-8x^2y^3)$ | 26. $(x - 8)(x - 7)$ |
| 17. $j^2(k^5j^3)$ | 27. $(6a + 1)(5a + 2)$ |
| 18. $a^4(b^4a^6)$ | 28. $(5x + 4y)(2x + 5y)$ |
| 19. $2x^3(9x^2 + 5y)$ | 29. $(2x + y)(4x - 9y)$ |
| 20. $5x^3(2x + 4y)$ | 30. $(6r - 5)(6r + 1)$ |
| 21. $5m^2(3m^3 + 5m^2 - 4m + 6)$ | 31. $(6c + 7)(6c - 7)$ |
| 22. $-4x^2y(x^2 + 7xy - 6y^3)$ | 32. $(3x + 5y)^2$ |
| 23. $(x + 6)(x + 2)$ | 33. $(x - 2)(x^2 - x + 3)$ |
| 24. $(x - 6)(x + 9)$ | 34. $(2x - 5)(5x^2 + 4x + 7)$ |

Divide the following polynomials:

- | | |
|--|---|
| 35. $\frac{9x-6}{3}$ | 45. $\frac{f^3+64}{f+4}$ |
| 36. $\frac{4x-7}{2}$ | 46. $\frac{4p-2+3p^2}{p-1}$ |
| 37. $\frac{x^2-3x+5}{x}$ | 47. $\frac{3m-4+2m^2}{m+5}$ |
| 38. $\frac{5x^2-25x+2}{-5x}$ | 48. $\frac{j^3-64}{j-4}$ |
| 39. $\frac{4x^{10}-5x^9-20x^4}{4x^2}$ | 49. $\frac{-5p+4p^2+4}{p-2}$ |
| 40. $(-x^6 + x^5 + 7x^2 - 9) \div x^4$ | 50. $(4p + 3p^2 - 1) \div (p + 4)$ |
| 41. $(x^2 + 2x + 6) \div x$ | 51. $(20x^2 - 13x + 2) \div (5x - 2)$ |
| 42. $(3x^2 - 15x + 5) \div (-3x)$ | 52. $(12x^2 - 6x^3 - 3 - 9x) \div (3x - 3)$ |
| 43. $(2x^{11} - 5x^7 - 10x^6) \div 2x^3$ | 53. $(8x^2 - 2x - 3) \div (2x + 1)$ |
| 44. $(-2x^6 + 5x^5 + 9x^2 + 2) \div x^4$ | 54. $(-3x^2 + 6x^3 - 4 - x) \div (2x + 1)$ |

POLYNOMIAL OPERATIONS PRACTICE ANSWERS

1. $12j^3 - j - 5$
2. $11a^5 + a - 6$
3. $8m^5 + 9m$
4. $12m^5 + 3m - 1$
5. $-8x^2 - 6x + 6$
6. $7x^3 - 7x - 2$
7. $2x^2 + x + 2$
8. $-4x^2 + 9x - 2$
9. $16x^2 - 8x + 5$
10. $6x^2 + 3x + 5$
11. $5x^3 - 6x + 3$
12. $-2x^3 + 10x^2 - 6x + 18$
13. $-x^3 + 11x^2 - 8x + 10$
14. $-2x^3 + 12x^2 - 8x + 10$
15. $-24x^5y^5$
16. $72x^5y^4$
17. j^5k^5
18. $a^{10}b^4$
19. $18x^5 + 10x^3y$
20. $10x^4 + 20x^3y$
21. $15m^5 + 25m^4 - 20m^3 + 30m^2$
22. $-4x^4y - 28x^3y^2 + 24x^2y^4$
23. $x^2 + 8x + 12$
24. $x^2 + 3x - 54$
25. $12x^2 - 29x + 15$
26. $x^2 - 15x + 56$
27. $30a^2 + 17a + 2$
28. $10x^2 + 33xy + 20y^2$
29. $8x^2 - 14xy - 9y^2$
30. $36r^2 - 24r - 5$
31. $36c^2 - 49$
32. $9x^2 + 30xy + 25y^2$
33. $x^3 - 3x^2 + 5x - 6$
34. $10x^3 - 17x^2 - 6x - 35$
35. $3x - 2$
36. $2x - \frac{7}{2}$
37. $x - 3 + \frac{5}{x}$
38. $-x + 5 - \frac{2}{5x}$
39. $x^8 - \frac{5x^7}{4} - 5x^2$
40. $-x^2 + x + \frac{7}{x^2} - \frac{9}{x^4}$
41. $x + 2 + \frac{6}{x}$
42. $-x + 5 - \frac{5}{3x}$
43. $x^8 - \frac{5x^4}{2} - 5x^3$
44. $-2x^2 + 5x + \frac{9}{x^2} + \frac{2}{x^4}$
45. $f^2 - 4f + 16$
46. $3p - 7 + \frac{5}{p-1}$
47. $2m - 7 + \frac{31}{m+5}$
48. $j^2 + 4j + 16$
49. $4p + 3 + \frac{10}{p-2}$
50. $3p - 8 + \frac{31}{p+4}$
51. $4x - 1$
52. $-2x^2 + 2x - 1 - \frac{2}{x-1}$
53. $4x - 3$
54. $3x^2 - 3x + 1 - \frac{5}{2x+1}$